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This research has been primarily concerned with the excitation and detection of inertial waves in a fluid contained in a rotating cylindrical cavity during spin-up of the fluid from rest. Application of this work is to the study of fluid filled projectiles and their stability in free flight. It is well known that a projectile containing fluid will be subject to instability if its nutational frequency is near one of the eigenfrequencies of the contained fluid. Our work has centered on finding these time dependent eigenfrequencies for a fluid which

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20. ABSTRACT CONTINUED

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AN EXPERIMENTAL STUDY OF INERTIAL WAVES IN A ROTATING FLUID CYLINDER DURING SPIN-UP OF THE FLUID FROM REST

Final Report

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Contents

1. Problem Studied

This research has been primarily concerned with the excitation and detection of inertial waves in a fluid contained in a rotating cylindrical cavity during spin-up of the fluid from rest. Application of this work is to the study of fluid filled projectiles and their stability in free flight. It is well known that a projectile containing fluid will be subject to instability if its nutational frequency is near one of the eigenfrequencies of the contained fluid. Our work has centered on finding these time dependent eigenfrequencies for a fluid which completely fills a cylindrical cavity. Also of interest in the application of our work is the rate at which these waves decay once the are excited during the spin-up period. Included in our experimental work was a study of decay rates for several inertial waves during spin-up from rest.

Our first problem was the excitation of inertial waves in a contained fluid while the fluid was spinning up from rest. Two techniques were devised as outlined in our research proposal to carry out this work. First we excited the waves in a partially filled cylindrical container by tilting the axis of symmetry of the container, which coincided with its rotation axis, at a small angle to the vertical. Inertial waves of azimuthal wavenumber unity at

frequency equal to the rotation speed were excited through the action of gravitational work on the viscously rotated center of mass. Resonance was determined in this case by the proximity of the fluid's aspect ratio to one of an infinite number of "eigenratios" which we calculated from classical theory at least for the inviscid, infinitessimal, steady state case. This method proved to be successful in exciting the inertial waves during the spin-up period.

Our next problem was to excite the inertial waves in a contained fluid in such a manner as to allow us to determine the decay rate as well as the eigenfrequency during spin-up from rest. This was accomplished by completely filling the cylindrical cavity with fluid and replacing the top end wall with a lid which could be made to precess at a frequency near the rotation speed. Since inertial waves of low order in their spatial complexity have frequencies in this range, a wave could be tuned in by the appropriate selection of the ratio of precession frequency to rotation speed for a given height-to-radius aspect ratio.

Measurement of the inertial wave response of the fluid required a quantitative method to detect the waves. In the experiments reported here and indeed in all the previous work on this type of problem we used the disturbance

pressure difference between two points in the fluid to obtain its response to our harmonic forcing.

Disturbance pressure measurements in the range of 1 mm of water in a system rotating at 1 hz led to some technical problems. For example, small fluctuations in the rotation rate of the container produced a disturbance pressure signal at the rotation frequency. Although fluctuations in rotation speed of the cylindrical container were necessarily kept to a minimum because the inertial waves themselves required this, some residual deviation in the rotation speed remained.

Precise control over the rotation speed of the container necessitated by the above considerations led to the design and construction of a feedback control system for the motor which drove the container. Such a controller was built for this purpose and the speed of the container was maintained to within 0.25% of its prescribed value throughout the duration of each experimental run.

In anticipation of the need for subsequent time series analysis of large sets of disturbance pressure data we built a digital data acquisition system and process controller for our 16 bit minicomputer. A 16 Channel analogue to digital converter (10 bit) was added to the

processor, floppy disc storage medium and hard copy printer. Thus our analogue output voltages from the pressure transducer could be converted to digital form for processing and storage.

Processing of our data was based mainly on a powerful technique to recover eigenfrequencies and decay rates from the disturbance pressure time sequences. This technique was developed in our laboratory and has recently found further application in the area of Applied Geophysics. Details of this method of data analysis can be found in a supplementary document referred to in the appendix of this final report.

2. Summary of Results

In the first part of this work inertial waves of azimuthal wavenumber unity were excited during spin-up from rest in a fluid partially filling a cylindrical cavity. In this study the axis of symmetry of the cylinder which coincided with its rotation axis was tilted at a small angle to the vertical. The upper surface of the fluid (silicone oil, kinematic viscosity 1 cs.) was free so that the viscously rotated centre of mass excited the wave. With this method of excitation, which was used by Thompson (1970), the wave frequency is always equal to the rotation speed; thus resonance occurred at certain critical filling ratios of depth to radius (eigenratios) of the cylinder. Waves were detected by disturbance pressure measurement with the points of observation on the bottom boundary of the container. The locations of these points were such that maximum pressure difference would be observed at steady state for the mode being studied. In this experiment interest centered on the mode with axial wavenumber 1, radial wavenumber 2 and azimuthal wavenumber 1. For short we refer to this as the (1,2,1) mode where the first number represents the axial wavenumber.

The dependence of eigenratio on time since the container began rotating is compared with a prediction by

Kitchens et al. (1978) in Figure 1. Shown in the Figure by the solid circles are points in time which corresponded to the maximum disturbance pressure observed during spin-up from rest. The time t, from switch on of the container's rotation is scaled by the spin-up time, T. Note that with the exponentiation in the ordinate time runs from top to bottom in the Figure. Since the Kitchens theoretical model differed from our experimental conditions some assumptions had to be made with regard to boundary conditions. In particular the free surface in the experiment was modelled in their calculation by a cylinder with end walls but with twice the height of the actual fluid in the experiment.

Several important differences have been observed between the resonant conditions in the above experiments on non-axisymmetric inertial waves and those on axisymmetric inertial waves studied previously by Aldridge (1975, 1977). Most significantly, as shown in Figure 2, the observed dependence of eigenratio on time since the container began rotating was a measurable function of the amplitude of perturbation which was in this case the tilt of the cylinder's axis. This clearly nonlinear behaviour was not found in the axisymmetric experiments. Not surprisingly, other nonlinear effects were in evidence in the tilted cylinder experiments. Probably the most important of these was the mean flow in the azimuthal

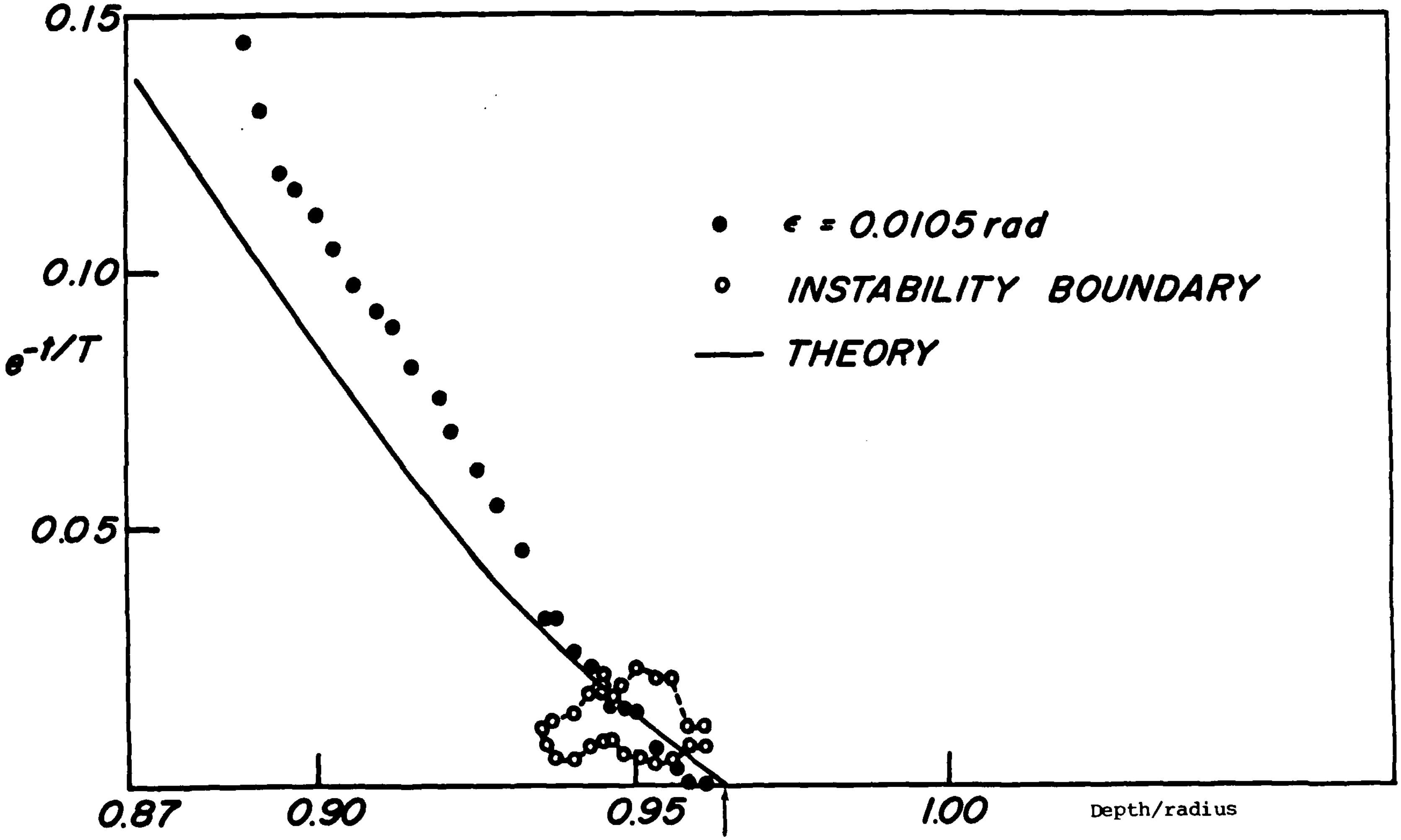

Figure 1. Time, t from switch on of container rotation to reach resonance of inertial wave (1,2,1). Times are divided by spin-up time T, and expressed in exponential form. Tilt is 0.0105 radians; rotation rate is 6.04 radians/second. Solid curve is predicted time dependence for eigenratio of (2,2,1) inertial wave. Open circles define instability region as determined from disturbance pressure records.

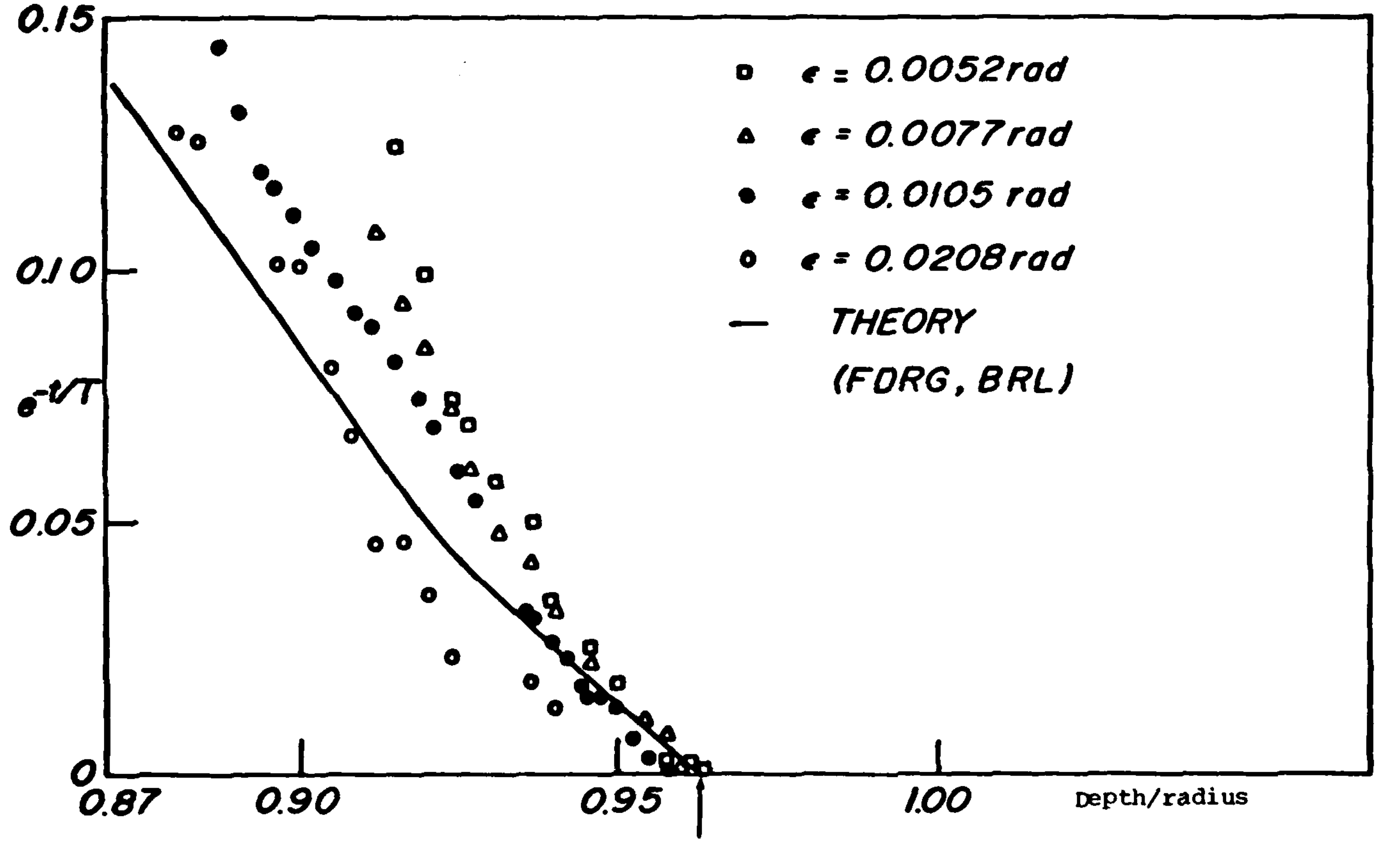

Figure 2. Nonlinear effect of tilt on the relationship between eigenfrequency and aspect ratio for (1,2,1) mode. The times t and T are as defined in Figure 1.

direction which developed both during spin-up and in the steady state. The mean flow itself would have been of no great interest to us since it is well known that such flows exist in this type of experiment. Unfortunately inertial forces in the mean flow were too great to be balanced by Ekman friction and the flow became unstable. At larger amplitudes of tilt it became impossible to adequately study the inertial waves and we detected the onset of this instability. Figure 1 shows the eigenratio dependence on time since the container was switched on and the onset of instability.

The existence of inertial wave resonance implies that should the forcing be removed the wave would decay. Thus a complementary study of the experiment outlined above would be the measurement of decay rates for the modes . At any point in time during spin-up a particular mode would have a certain decay rate and frequency which we term the complex eigenfrequency. Measurement of these quantities was accomplished by switching off the perturbation after a wave had been excited. This simple procedure allowed the direct measurment of both frequency and decay rate at any point in time during spin-up. It is relevant to note here that these measurements on decay rate were motivated by requests from the Fluid Dynamics Research Group at B.R.L. for such information.

Development of a new data analysis technique made possible the simultaneous recovery of both frequency and decay rate of inertial oscillations. The disturbance pressure time sequence was modelled as a decaying exponential times a sinusoid. By minimizing the square of the difference between the data and the expected response with respect to each of the model parameters, estimates of these parameters were found. Our procedure was necessarily an iterative one since the model is nonlinear in the parameters. Thus updates to estimates of the parameters are found at each step of the iteration process. Included with each estimate of a parameter is its error deviation as found in the minimization process.

Our method was initally tested by recovering both frequency and decay rate from numerically simulated experimental runs. These proved to be successful in returning the correct complex eigenfrequency complete with error estimates corresponding to the pseudo random noise that we added to the numerical simulation.

The first use of our method on real data came from a repetition of one of the earlier experiments by Aldridge (1977) on the excitation of axisymmetric inertial waves (inertial oscillations) of a fluid in a cylindrical cavity

during spin-up from rest. Shown in Figure 3 by the solid circles are the recovered eigenfrequencies for the (2,1,0) mode during spin-up of the fluid from rest. Time is scaled as in Figures 1 and 2 while the abscissa in this case is the actual rotation speed of the container divided by the eigenfrequency as recovered by our inversion method. The solid line in the Figure represents a prediction from a theoretical model by Lynn (1973).

Decay rates for this mode as well as eigenfrequencies for several other modes were measured in this manner and are shown in table 1. Decay rate for the (2,1,0) inertial oscillation in the state of solid body rotation is in reasonable agreement with the calculated one as shown in Table 1 . Our calculation is based on a model for dissipation due to Kudlick (1966). As expected decay rates during spin-up from rest are greater than in the state of solid body rotation. No decay rates can be obtained for the tilted cylinder experiments because this method of excitation precludes such measurements.

Our latest series of experiments on the excitation of inertial waves using a rotating precessing lid, as introduced by McEwan (1970), allowed us to measure the full complex eigenfrequency during spin-up from rest. In this study a fluid which completely filled a cylindrical cavity

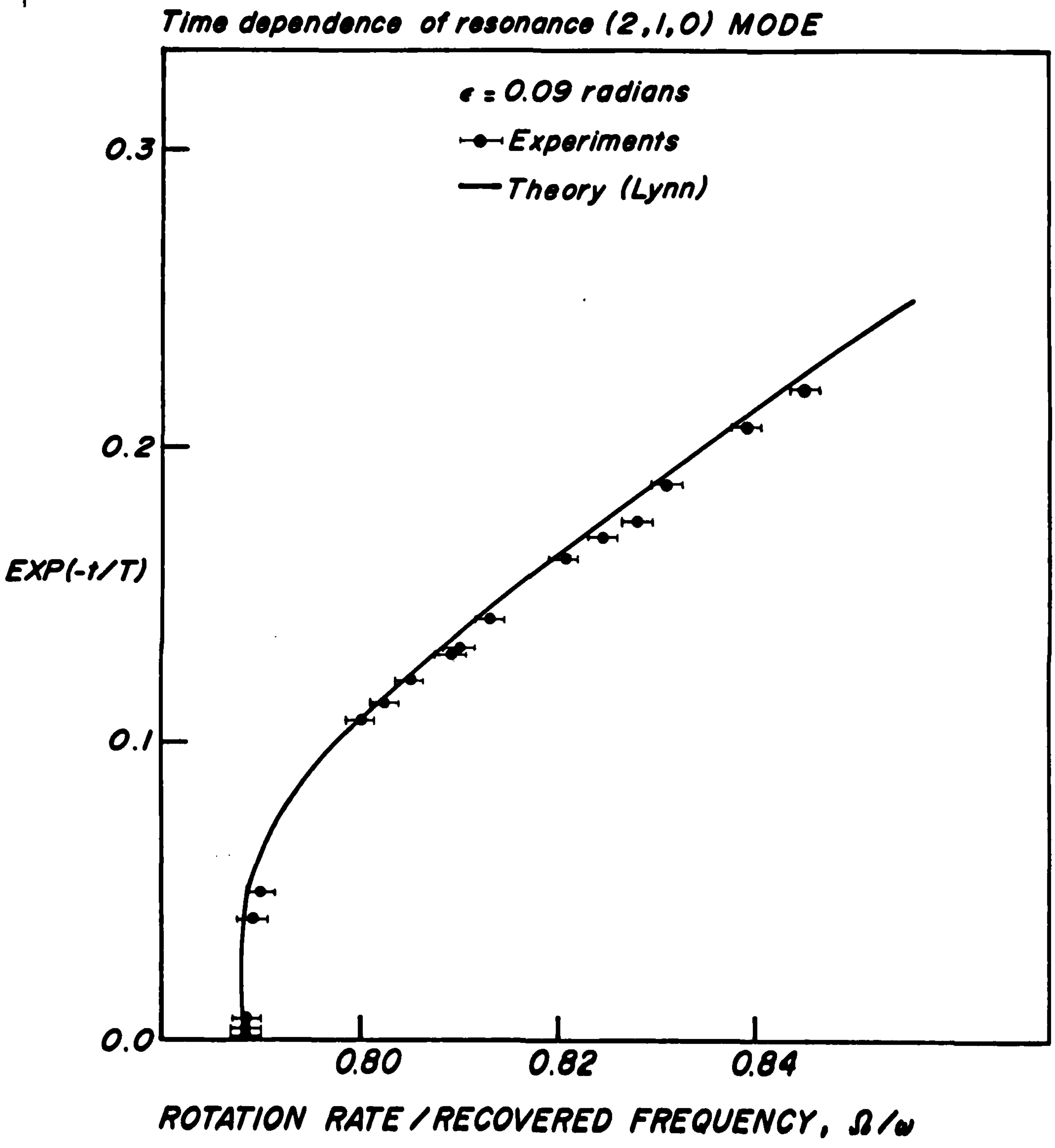

Figure 3. Eigenfrequency as a function of time t since switch on of the container's rotation. Time is scaled with the spin-up time T and the abscissa represents solid body rotation.

	t/T	$S_{210}(t)$	$(\frac{\Omega}{\omega})_{210}(t)$	$(\frac{\Omega}{\omega})_{420}(t)$	$(\frac{\Omega}{\omega})_{850}(t)$	$(\frac{\Omega}{\omega})_{630}(t)$
THEORY	∞	2.770	0.788	0.749	0.824	0.734
EXPERIMENTS	4.75	2.48 $\pm$ 3.0%	0.788 $\pm$ 0.11%	0.749 $\pm$ 0.6%	0.823 $\pm$ 1%	0.716 $\pm$ 0.6%
	3.04	2.55 $\pm$ 3.2%	0.790 $\pm$ 0.11%		0.816 $\pm$ 1.7%	0.725 $\pm$ 1.4%
	2.02	2.78 $\pm$ 6.0%	0.809 $\pm$ 0.13%		0.812 $\pm$ 0.5%	
	1.68	4.16 $\pm$ 3.0%	0.832 $\pm$ 0.15%			
	1.51	4.45 $\pm$ 3.5%	0.845 $\pm$ 0.20%			

Table 1. Recovered dimensionless eigenfrequencies and decay rates for axisymmetric modes during spin-up from rest and predicted values from linear inviscid theory for solid body rotation.

had its top boundary precessing at a small angle all the while the fluid was spinning up from rest. Several inertial waves have been excited and their frequencies and decay rates have been measured. Shown in Figure 4 by the solid circles are recovered eigenfrequencies for the (1,2,1) mode during spin-up. The triangles in this Figure are the recovered forcing frequencies of the precessing lid. Clearly the system rings at its natural frequency whether approached from above or below. The squares in this Figure represent the eigenfrequency that would have been obtained had we used peak disturbance pressure response to define resonance during spin-up.

For comparison we show in Figure 5 the measured eigenfrequencies of the (1,1,1) mode during spin-up from rest. The opposite dependence of eigenfrequency on time for this mode is at least in part reconciled by the prediction of linear theory that the eigenfrequency should tend to the rotation rate at zero, i.e. switch on, time. From the point of view of an observer in a fixed frame of reference this means zero frequency. We have recently compared the results shown in Figures 4 and 5 with eigenfrequencies calculated throughout spin-up by Sedney (1981) and found agreement to within 2%. Most significantly these experiments confirmed Sedney's expectation that viscous corrections for endwall effects

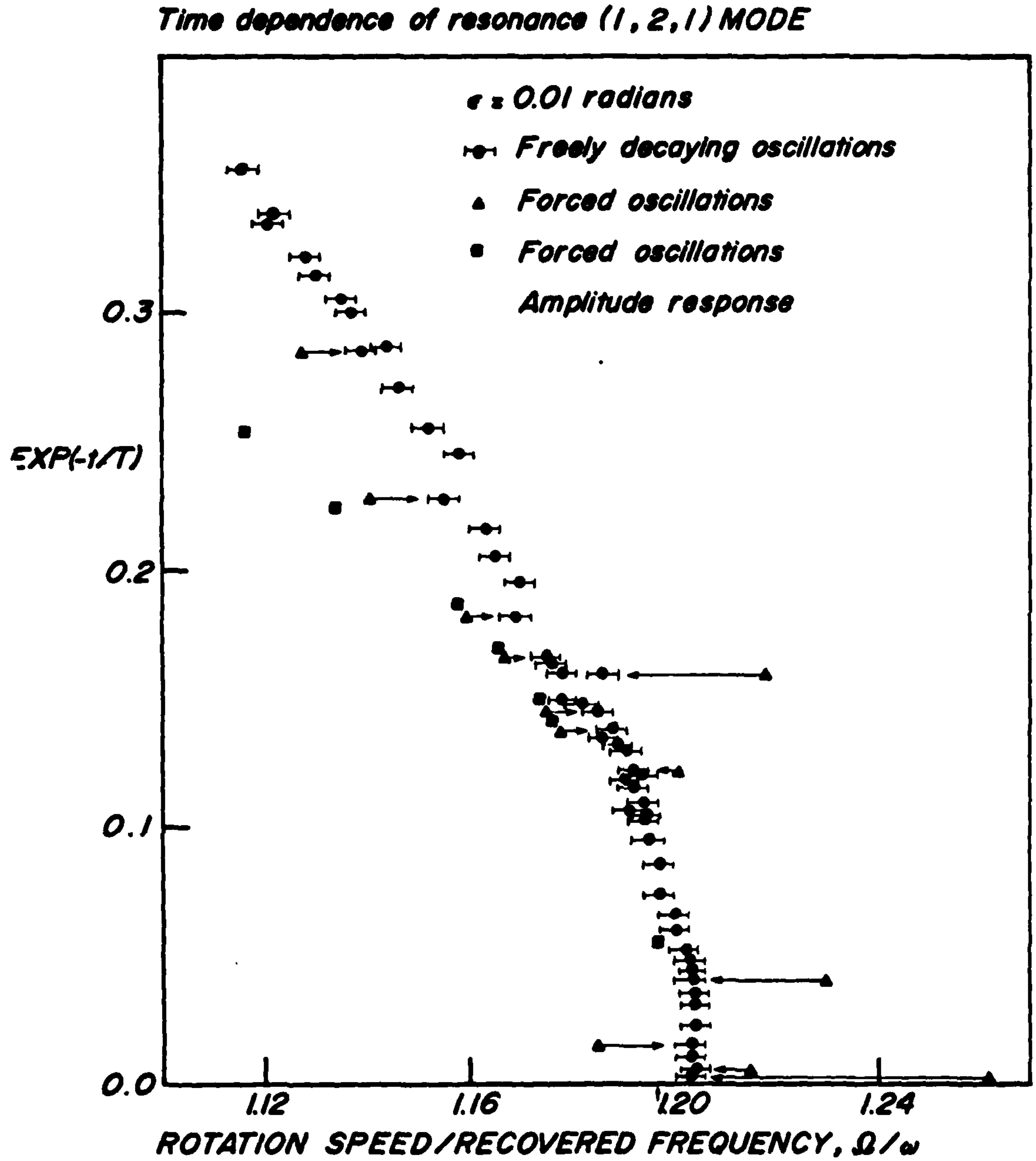

Figure 4. Recovered eigenfrequencies for the (1,2,1) mode during spin-up from rest are shown by solid circles with error bars from inversion of disturbance pressure records. Triangles represent forcing frequency prior to switch off at time t of the precession of amplitude 0.01 radians. Small arrow on the abscissa is predicted eigenfrequency from invscid linear theory for solid body rotation.

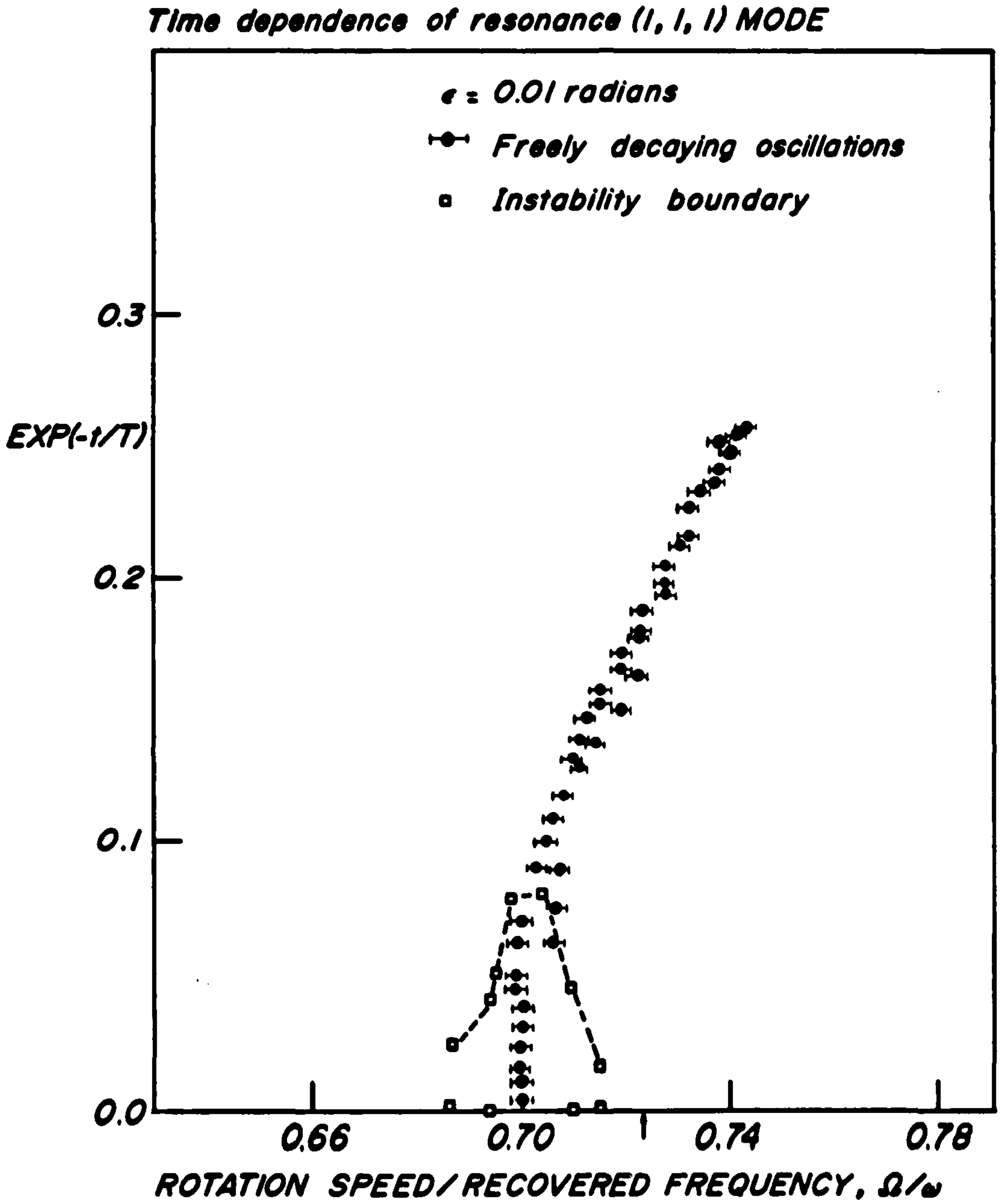

Figure 5. Recovered eigenfrequencies for the (1,1,1) mode during spin-up from rest are shown by solid circles with error bars from inversion of disturbance pressure records. Open circles correspond to the onset of the instability. Small arrow on the abscissa is predicted eigenfrequency from inviscid linear theory for solid body rotation.

were unnecessary. Our experiments complement a related set of experiments by Whiting (1980) who excited inertial waves in a rotating cylinder in solid body rotation by precessing the whole cylinder at a frequency near one of the fluid's eigenfrequencies. He found that nonlinear effects appeared at much lower Rossby Number (precession amplitude) than in our experiments.

Our precessing lid results have been complicated by some nonlinear effects which were to be expected from our previous experience with the tilted cylinder study. Again we found the mean flow instability but the first onset has occurred at larger amplitude presumably because the upper boundary supplies additional Ekman friction. The open squares in Figure 5 show the onset of the instability.

The greatest complications of nonlinear effects arose in the free decay of the inertial waves. When the precessing lid was stopped the inertial wave did not simply decay as our rather simple model expected. Instead several other waves, which had their frequencies sufficiently close to the one being excited by the precessing lid, decayed simultaneously. This mix of waves was partially sorted out by simply adding other modes to our model. It became apparent, however, that rather than just a superposition of modes our pressure time sequence represented an interaction

amoung the waves detected since individual waves not present in the initial portions of the sequence appeared later. Such an interaction is of course not surprising in view of the previously mentioned nonlinear features of the fluid's response.

Separation of the data into overlapping intervals allowed us to recover instantaneous decay rates during ringdown of the inertial waves while the fluid was spinning up from rest. Shown in Figure 6 are the measured values of decay rate presented in dimensionless form S(t), for the (1,1,1) and (1,2,1) modes. The abscissa represents time since the container's rotation began scaled in terms of the spin-up time, T. Theoretical values of the dimensionless decay rates in the steady state for the two modes were calculated from Kudlick's model and are displayed at the right of the Figure. Clearly decay rates are less well determined than the eigenfrequencies as would be expected. In spite of this a resonably well defined dependence of decay rate on time since the container began rotating can be seen in the Figure. In general we would expect the decay rates to be larger closer to switch on but we do not even begin to speculate why the two modes behave so differently in this regard.

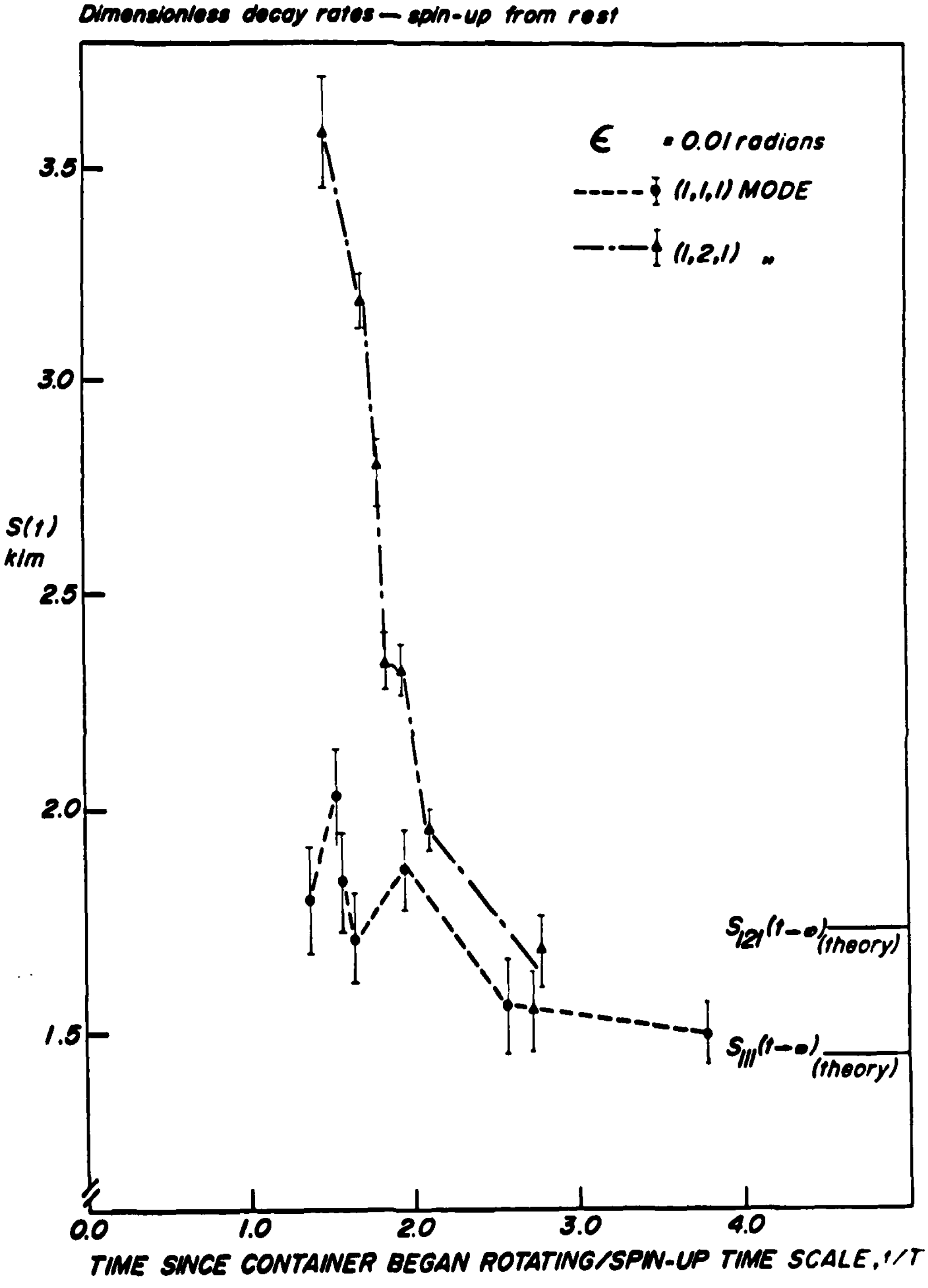

Figure 6. Dimensionless decay rates during spin-up from rest. Dimensional decay rates are obtained by dividing the values shown by the spin-up time, T where T = 57 seconds and T = 45 seconds for the (1,1,1) and (1,2,1) modes respectively.

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5. Scientific Personnel

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6. Appendix

Further technical details and all other results of the work carried out under this contract are available as Report 781 of the York University Hydrodynamics Laboratory. This report contains expanded versions of the paper by Stergiopoulos and Aldridge (1980) and the paper by Aldridge and Stergiopoulos (1981) presented in May of this year and currently in preparation for publication. Reproduced below is a list of its contents.